This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6715**

Unique Paper Code : **32371303** **HC**

Name of the Paper : **Mathematical Analysis**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt in all *four* questions from Section I

and *three* questions from Section II.

Question Nos. **1** and **6** are compulsory.

Use separate answer-books for Section I and Section II.

Section I

1. (*a*) Write the Supremum and Infimum of the following :

$$S = \left\{1 - \frac{(-1)^n}{n} : n \in N\right\}.$$

(*b*) State whether the following statements are Ture *or* False :

(*i*) The set Z of integers has infinite limit points

(*ii*) Every Cauchy sequence is convergent.

(*c*) Is every bounded sequence convergent ? Justify.

(*d*) Is the sequence $<a_n>$ defined as $a_n = \frac{(-1)^n}{n}$ bounded ? Justify.

(*e*) Give the geometrical interpretation of Rolle's theorem. 5×2

2. (*a*) Define an open set. Show that the union of an arbitrary family of open sets is also an open set.

(*b*) Show that the set $S = \{1/n : n \in Z^+\}$ has only one limit point. 6,6

3. (*a*) Show that the sequence $<a_n>$ defined by :

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}, n \in N$$

converges.

(*b*) Let f be the function defined on [0, 1] by setting :

$$f(x) = (-1)^r, \text{ when } \frac{1}{r+1} \le x < \frac{1}{r}, r = 1, 2, 3, ...,$$

$f(0) = 0$ and $f(1) = 1$.

Examine for continuity of the function f at the points

$x = 1, \frac{1}{2}, \frac{1}{3}, ..., \frac{1}{r}, ..., 0.$ 6,6

4. (*a*) Let Σu_n be a series of positive terms with $\lim_{n\to\infty} u_n^{1/n} = l$. Prove that if $l < 1$, then Σu_n converges. What happens, if $l = 1$?

(*b*) (*i*) Test for convergence of the series whose *n*th term is $\sqrt{n^3+1} - \sqrt{n^3}$.

(*ii*) Show that the sequence $<a_n>$ where

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}; \forall n \in N,$$ does not converge. 6,6

5. (*a*) State Lagrange's theorem on $[a, a + h]$. Prove that for any quadratic function $px^2 + qx + r$, the value of θ in Lagrange's theorem is always $\frac{1}{2}$, whatever p, q and r may be.

(*b*) Obtain Maclaurin's series expansion of $(1 + x)^m$, where m is a positive integer. 6,6

Section II

6. (*a*) For interpolation of $f(x)$ relative to 0, α, 1; if Lagrange's formula gives the result as :

$$f(x) \equiv \left[1 - \frac{x(x-\alpha)}{1-\alpha}\right] f(0) + \frac{x(1-x)}{1-\alpha} .$$

$$\frac{f(\alpha) - f(0)}{\alpha} + \frac{x(x-\alpha)}{1-\alpha} . f(1),$$

then what is the approximate result for $f(x)$ when $\alpha \to 0$.

(*b*) What is the value of $\Delta^2 f(x)$? Where $f(x) = x^2 + 4x + 9$.

(*c*) Represent $f(x) = x^3 + 6x$ in terms of factorial notations.

(*d*) Out of total 3 Cote's numbers, the last Cote's number has a value equals to 1/6. Write the values of the remaining two Cote's numbers.

(*e*) $\sqrt{\left(1 + \frac{1}{4}\delta^2\right)} \equiv$ ......, where δ is central difference operator. 1×5

7. (*a*) If u_x be a function whose differences when the increment of x is unity, are denoted by $\delta u_x, \delta^2 u_x$, ... and $\Delta u_x, \Delta^2 u_x$, ... when the increment of x is n, then if $\delta^2 u_x, \delta^2 u_{x+1}, \delta^2 u_{x+2}$ ...... are in G.P. with common ratio q, show that :

$$\frac{\Delta u_x - n\delta u_x}{(q^n - 1) - n(q-1)} = \frac{\delta^2 u_x}{(q-1)^2}.$$

(*b*) Prove that :

$$u_1 + u_2 + \ldots\ldots + u_n = \binom{n}{1} u_1 + \binom{n}{2} \Delta u_1 + \binom{n}{3} \Delta^2 u_1 + \ldots\ldots + \Delta^{n-1} u_1 .$$

6,6

8. (*a*) Derive Lagrange's interpolation formula. Show that sum of the Lagrangian coefficients is unity.

(*b*) Show that :

$$\sum_{k=0}^{n-1} \delta^2 f_{2k+1} = \tanh \frac{U}{2}\left(f_{2n} - f_0\right),$$

where U and δ have their usual meaning. 6,6

9. (*a*) State and prove Newton-Cote's integration formula and hence show that Cote's numbers are symmetric.

(*b*) Solve any *two* of the following difference equations :

(*i*) $u_{x+2} - 4u_x = 9x^2$

(*ii*) $u_{x+2} - 7u_{x+1} - 8u_x = x^{(2)} 2^x$

(*iii*) $u_{x+1} = 2u_x \sqrt{1 - u_{x^2}}$. 6,6

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6713**

Unique Paper Code : **32371301** **HC**

Name of the Paper : **Sampling Distribution**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory.

Attempt *six* questions in all by selecting at least *two* questions from each section.

1. Attempt any *five* parts : 5×3=15

(*a*) Define convergence in distribution and convergence in probability and state their relations.

(*b*) Discuss type-I and type-II errors and level of significance with examples.

(*c*) Decide whether the central limit theorem holds for the sequence of independent random variables X_k with distribution defined as $P(X_k = \pm k^{\alpha}) = 1/2$.

P.T.O.

(*d*) Show that the sum of independent Chi-square variates is also a χ^2 variate.

(*e*) If $X \sim F_{2,4}$, then show that :

$$P(X \geq 2) = 1/4.$$

(*f*) If $X \sim F_{m,n}$ and $Y \sim F_{n,m}$, then show that :

$P(X \leq a) + P(X \leq 1/a) = 1$ for all a.

(*g*) In a 2 × 3 contingency table, if $N = x + y + z$, $N' = x' + y' + z'$ and $N = N'$ then show that :

$$\chi^2 = \frac{(x - x')^2}{x + x'} + \frac{(y - y')^2}{y + y'} + \frac{(z - z')^2}{z + z'} \sim \chi^2_2.$$ 5×3

Section A

2. (*a*) If X is a random variable and $E(X^2) < \infty$, then prove that $P(|x| \geq a) \leq E(X^2)/a^2$, for all $a > 0$. Use Chebychev's inequality to show that for $n > 36$ the probability that in n throws of a fair die, the number of sixes lies between $\frac{n}{6} - \sqrt{n}$ and $\frac{n}{6} + \sqrt{n}$ is at least 31/36.

(*b*) If $X_1, X_2, \ldots\ldots, X_n$ are iid random variables with mean μ_1 and variance σ_1^2 (finite) and $S_n = X_1 + X_2 + \ldots\ldots + X_n$, then :

$$\lim_{n \to \infty} P[a \leq \frac{S_n - n\mu_1}{\sigma_1 \sqrt{n}} \leq b] = \varphi(b) - \varphi(a), \text{ for } -\infty < a < b < \infty,$$

where $\varphi(.)$ is the distribution function of a standard normal variate. 6,6

3. (*a*) Let $\{X_n\}$ be a sequence of mutually independent random variables such that $P(X_n = \pm 1) = \frac{1 - 2^{-n}}{2}$ and $P(X_n = 0) = 2^{-n}$. Examine whether the weak law of large numbers can be applied to the sequence $\{X_n\}$.

(*b*) Given a random sample of size n from exponential distribution :

$$f(x) = \alpha e^{-\alpha x}, x \geq 0, \alpha > 0.$$

Show that $X_{(r)}$ and $W_{rs} = X_{(s)} - X_{(r)}$, $r < s$, are independent. Also find the distribution of $X_{(r+1)} - X_{(r)}$. 6,6

4. (*a*) Derive the expression for the standard error of :

(*i*) the mean of a random sample of size n.

(*ii*) the difference of the means of two independent random samples of size n_1 and n_2.

(*b*) P_1 and P_2 are the (unknown) proportions of students wearing glasses in two universities A and B. To compare P_1 and P_2, samples of size n_1 and n_2 are taken from the two populations and the number of students wearing glasses is found to be x_1 and x_2 respectively. Suggest an unbiased estimate of $P_1 - P_2$ and obtain its sampling distribution when n_1 and n_2 are large. Hence explain how to test the hypothesis $H_0 : P_1 = P_2$ against $H_1 : P_1 \neq P_2$. 6,6

Section B

5. (*a*) Obtain mean deviation about mean of t-distribution with n d.f.

(*b*) If X is a Chi-square variate with n *df*, then prove that for large n :

$$\sqrt{2X} \sim N(\sqrt{2n}, 1)$$

(*c*) Show that t-distribution tends to normal distribution for large n. 4,4,4

6. (*a*) For a Chi-square distribution with n d.f., prove that :

$$\mu_{r+1} = 2r(\mu_r + n\mu_{r-1}), r \geq 1.$$

Hence find β_1 and β_2. Also discuss the limiting form of χ^2 distribution.

(*b*) If $X \sim F_{m,n}$ distribution, obtain the distribution of mX when $n \to \infty$. Also obtain the mode of the F-distribution. 6,6

7. (*a*) Prove that if $n_1 = n_2$, the median of F-distribution is at F = 1 and that the quartiles Q_1 and Q_3 satisfy the condition $Q_1Q_3 = 1$.

(*b*) Discuss the t-test for testing the significance for the difference of two population means. 6,6

8. (*a*) Let $X_1, X_2, \ldots\ldots\ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ and $\bar{X}$ and S^2 respectively be the sample mean and sample variance. Let $X_{n+1} \sim N(\mu, \sigma^2)$, and

assume that $X_1, X_2, \ldots\ldots, X_n, X_{n+1}$ are independent. Obtain the sampling distribution of :

$$U = \frac{x_{n+1} - \bar{X}}{S}\sqrt{\frac{n}{n+1}}.$$

(*b*) If $X \sim F_{n_1, n_2}$, then show that its mean is independent of n_1.

(*c*) If X is Poisson variate with parameter λ and χ^2 is a Chi-square variate with 2K d.f., then prove that for all positive integers k :

$$P(X \leq k - 1) = P(\chi^2 > 2\lambda).$$ 4,4,4

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7128**

Unique Paper Code : **32373902** HC

Name of the Paper : **Statistical Data Analysis Using R**

Name of the Course : **B.Sc. (Hons.) Statistics : SEC**

Semester : **III**

Duration : **2 Hours** Maximum Marks : **50**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions from Section A and *two* questions each from Section B and Section C. Write R codes for each question given in Section B and C along with other question related answers.

Section A

1. (*a*) CRAN in R stands for Comprehensive R ....

(*b*) A command used to extract 6th element from a vector x to 8 elements is ....

(*c*) For a given vector $x = c(3, 1, 2, 5, 4, 8, 9, 5)$, the values obtained by using cummax(x) are ....

(*d*) A command/R code abline (v = value) is used for drawing .... line.

(*e*) Graphical window can be divided into several parts using the graphical instruction .... 5×1

2. (*a*) Can we use customized x-axis limit and y-axis limits in a graphical representation. Give example.

(*b*) Write R codes to obtain $P(X \leq 3)$, where X ~ *Binomial* (n = 12, *prob.* = 0.4).

(*c*) Write the output of the following R codes :

```
X <- seq(10, 70, 20)

X
```


(*d*) Write the arguments used in graphical representation of R for the line type and line width.

(*e*) What are the differences in high level and low level plots and name one each of high level and low level plot ? 5×2

Section B

3. Given the frequency distribution $x_i|f_i$, draw less than and more than ogives in a single plot. 7.5

4. Draw a histogram for a grouped frequency distribution with equal class intervals. 7.5

5. (*i*) For a given vector, draw a pie chart with the initial angle 90 degrees and it is in clockwise direction.

(*ii*) Draw a random sample of size 25 from normal distribution with mean 5 and variance 1 and draw either the box plot or spike chart. 7.5

Section C

6. For a given raw data, obtain the grouped frequency data with 6 class intervals. Also obtain the mid value for each class and the cumulative frequencies. 10

7. Fit a Poisson distribution for given $x_i|f_i$, (i = 1, 2, ..., 6) and also test the goodness of fit. 10

8. Write R-code for t-test for difference of means when the samples are drawn from same population. Also interpret the results as obtained in R. Further write R codes for mean, variance, median and mode for both the samples used in the above t-test. 10

This question paper contains **8** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7127**

Unique Paper Code : **32373901** **HC**

Name of the Paper : **Statistical Data Analysis Using Software Packages**

Name of the Course **B.Sc. (H) Statistics : SEC**

Semester : **III**

Duration : **2 Hours** Maximum Marks : **50**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt questions from any *one* of the two Parts I or II.

All questions are compulsory.

Part I (Based on SPSS)

1. Fill in the blanks : 5×1

   (*i*) The default variable type is ....

   (*ii*) SPSS variable names are limited to .... characters.

   (*iii*) The variable labels can be up to .... characters long.

   (*iv*) The value label can be up to .... bytes.

   (*v*) The data editor has two views .... and ....

P.T.O.

2. Write SPSS procedure for the following (answer any *five*) : 5×2

(*i*) Insert new variables between any *two* pre-defined variables.

(*ii*) Change column width once it has been declared.

(*iii*) Construct box-plot.

(*iv*) To create a date variable.

(*v*) To create histogram with overlaid normal curve.

(*vi*) To create frequency distribution from given raw data.

3. Answer the following : 5×1

(*i*) When performing CROSSTABS can control variables be added to the procedure ?

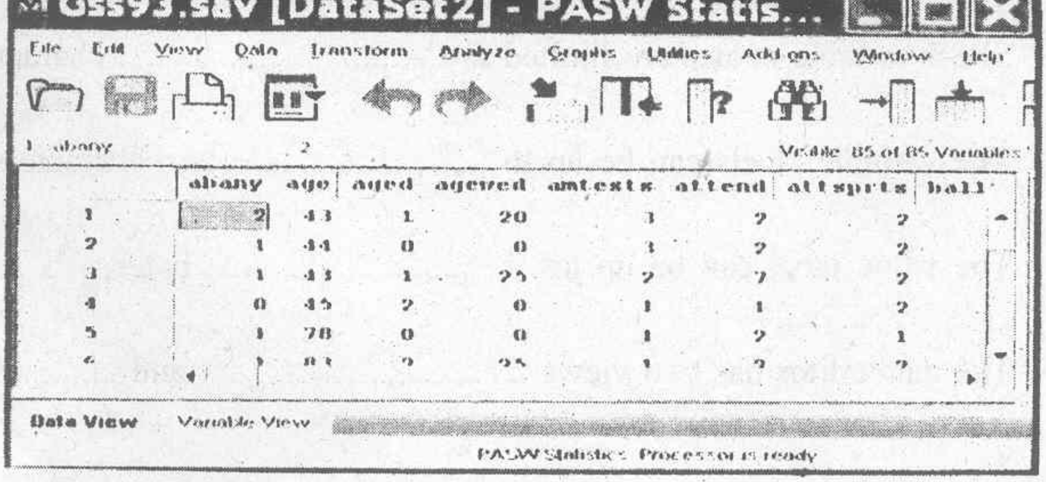
(*a*) Yes, by placing the variables in the row box.

(*b*) Yes, by placing the variables in the column box.

(*c*) Yes, by placing the variables in the layer box.

(*d*) No, control variables cannot be incorporated in a cross tabulation.

(*ii*) After studying the figure below, select the *correct* statement :



(*a*) Gss93.sav is the dataset name.

(*b*) DataSet2 is the dataset name.

(*c*) DataSet2 is the file name.

(*d*) DataSet2 is both the file and dataset name.

(*iii*) What is the main advantage of using syntax ?

(*a*) It can be run using the menus.

(*b*) It is the most popular way of running IBM SPSS Statistics.

(*c*) It can be saved and retrieved for subsequent analyses.

(*d*) It is the same format as syntax in other statistical software.

(*iv*) Using the Count Values within Cases transformation, a user can specify the following combinations of values to count (select all that apply) :

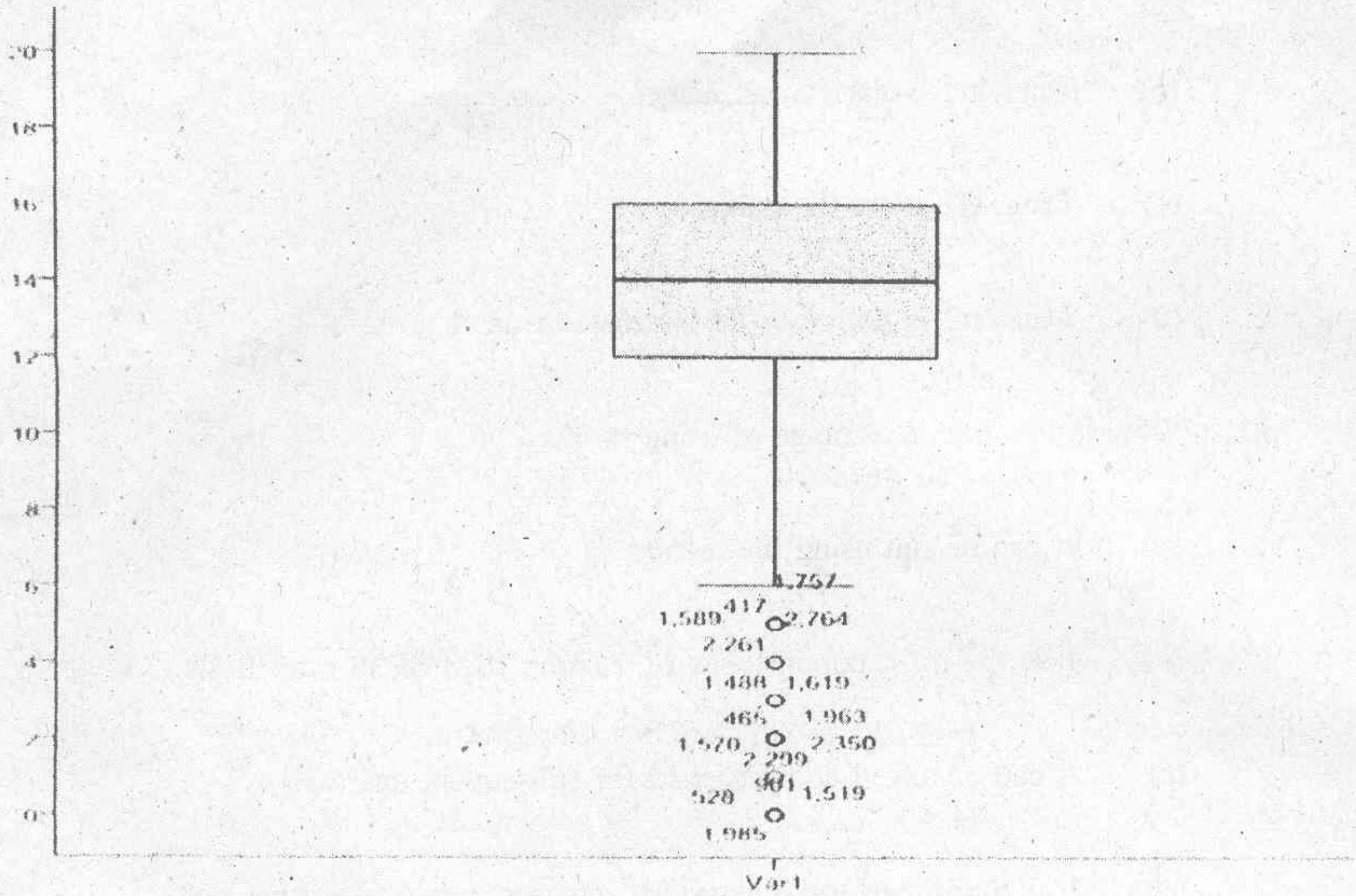
(*a*) Range of values

(*b*) System-missing only

(*c*) User-missing only

(*d*) Individual values

(*v*) Which statement is *true* about this box plot ?



(*a*) The mean is 14.

(*b*) The standard deviation is 14 (20 minus 6).

(*c*) The standard deviation is 4 (16 minus 12).

(*d*) All of the outliers are on the lower end of the distribution.

4. Answer any *six* of the following : 6×5

(*i*) Give procedure to generate summary statistics and graphical displays for the :

(*a*) Categorical data,

(*b*) Scale variable.

(*ii*) State the utility of 'Select cases' option with the help of an example.

(*iii*) Briefly, discuss the attributes which can be defined in variable view.

(*iv*) Discuss the output window of the option 'Descriptives' in SPSS.

(*v*) Using 'recode function' in SPSS, write the procedure for constructing frequency distribution from the given raw data in case of :

(*a*) equal class intervals,

(*b*) unequal class intervals.

(*vi*) Describe basic structure of an SPSS data file.

(*vii*) Name any *three* interfaces of SPSS and give their applications.

Part II (Based on MATLAB)

1. Attempt any *ten* of the following :

10×3

Consider the following to attempt the questions :

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 5 & 7 & 9 \\ 1 & 8 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 8 \\ 3 \end{bmatrix} \quad a = [6 \quad -5 \quad 7]$$

(*i*) What is the difference between A^2 and A.^2 ? Explain.

(*ii*) What is the result of executing *a*.^2 and *a*^2 ?

(*iii*) What is the command that extracts or isolates 2nd and 3rd rows of the matrix A ?

(*iv*) What is the result of A < *a* ?

(*v*) What is the result of *diag*(A) ?

(*vi*) Explain the commands *close, dir, cd* with *one* example each.

(*vii*) Write the general form of *path*(...). Give *one* example.

(*viii*) What is the output of the following code segment ?

```
n = 4; i = 0; s = –10;
while i <= n;
s = s + 1;
i = i + 1;
end
[i s]
```

(*ix*) What is workspace in MATLAB ? Explain commands *who, whos*.

(*x*) Fill in the blanks :

(*a*) A variable declared outside of all functions is called ....

(*b*) The number of input arguments to a function is returned by ....

(*c*) The function *linspace*($x1$, $x2$) generates a row vector of .... linearly equally spaced points between $x1$ and $x2$.

(*xi*) Write a function to evaluate the following such that it may be called with a vector argument :

$$f(x) = \begin{cases} 3x^2 - 2x + 6, & \text{for } x < 0; \\ 4x^2 - 3x + 5, & \text{for } 0 \le x \le 2; \\ 4x + 17, & \text{for } x > 2. \end{cases}$$

(*xii*) Given $x = [5, 2, 6, 7, 3, 4, 8]$, what will be the value of y for the following relations :

(*a*) $y = x > 5$

(*b*) $y = x(find(x < 5))$

(*c*) $y = length(find(x > 5))$.

2. Attempt any *two* of the following : 2×4

(*i*) Explain with suitable examples, the *fprintf*(...) function used for displaying formatted data on screen.

(*ii*) Discuss a method for drawing random samples from a distribution having c.d.f. $F(x) = 1 - e^{-\alpha x^{\beta}}$ using the inverse transformation method.

(*iii*) Write a *for* loop that computes the sum $1 + \frac{1}{x} + \frac{1}{x^2} + ... + \frac{1}{x^n}$ for user input values of n and x.

3. Attempt any *one* of the following : 1×5

(*i*) What will be the output of the following program ? Explain.

```
n=6;
n1=1;
n2=1;
y=[n1 n2];
for i=1:n
        y
        x=n1+n2;
        n1=n2;
        n2=x;
        y=[y x];
end
```

(*ii*) Write the functions, in general, you would use to compute 'pdf', 'cdf' and 'rnd' for the :

(*a*) Beta distribution,

(*b*) Normal distribution and

(*c*) Uniform distributions.

4. Attempt any *one* of the following : 1×7

(*i*) Write a function that will generate Chi-Square random variables with *n* degrees of freedom by generating *n* standard normal, squaring them and then adding them up. Hence write the program to generate 100 r.v.s. and calculate mean, variance.

(*ii*) Write a program to generate a sample of 120 observations from the normal distribution with mean $\mu = 52.2$ and variance $\sigma^2 = 67.24$. Using the sample drawn, estimate the mean, variance and the skewness in the population. Also, plot the probability histogram along with the superimposed normal curve.

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6714**

Unique Paper Code : **32371302** **HC**

Name of the Paper : **Survey Sampling and Indian Official Statistics**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all,

selecting *three* from each Section.

Section I

1. (*a*) Define simple random sampling without replacement from a finite population. Derive the unbiased estimator of the population mean and find its sampling variance.

(*b*) For srswor, prove that :

$$\text{cov}(x_i, \bar{y}_n) = \frac{N-n}{Nn} \cdot \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X}_N)$$

$$(Y_i - \bar{Y}_N) = \frac{N-n}{n(N-1)} \text{cov}(X, Y)$$

Also evaluate $E(\bar{x}_n, \bar{y}_n)$. 6,6½

P.T.O.

2. (*a*) Compare regression estimator with ratio estimator and simple random sample mean, assuming the formulae for the variances of the estimators.

(*b*) Prove that the mean of cluster means $\bar{\bar{y}}$ is an unbiased estimator of population mean with variance given as :

$$V(\bar{\bar{y}}) = \frac{N-n}{N-1} \cdot \frac{\sigma^2}{nM} [1 + (M-1)\rho].$$ 5,7½

3. (*a*) Obtain the estimated relative efficiency of cluster sampling with respect to srswor.

(*b*) Define difference estimator and derive from it the regression estimator. Also obtain the variance of regression estimator under first approximation. 5,7½

4. (*a*) If y and x are unbiased estimators of the population totals of Y and X respectively, show that the variance of ratio estimate $\frac{y}{x}$ can be approximated by $C_y^2 - C_x^2$, where c_x and c_y are coefficient of variation of x and y respectively. (The correlation coefficient between $\frac{y}{x}$ and x is assumed to be negligible).

(*b*) From a simple random sample of size n drawn from N units by srswor, a simple random sub-sample of n_1 units is duplicated and added to the original sample. Show that the mean based on $(n + n_1)$ units is an unbiased estimator of the population mean. Also obtain its variance. How does it compare with the variance of the estimator based on n units only. 6,6½

Section II

5. (*a*) Derive the variance of the estimate of the population mean based on systematic sampling in terms of intra-class correlation coefficient ρ. Prove that reduction in this variance over srswor will be 100% if ρ assumes the minimum possible value. If ρ assumes the maximum value, what is the relative efficiency of systematic sampling over simple random sampling ?

(*b*) Justify the following statements :

(*i*) The smaller the size of stratum, the smaller should be the size of sample to be selected therefrom.

(*ii*) The smaller the variability within a stratum, the smaller should be the size of sample selected from the stratum.

(*iii*) The cheaper the cost per unit in a stratum, the larger should be the size of sample selected from that stratum.

Hence obtain minimum size required for estimating population mean with fixed variance under optimum allocation. 6½,6

6\. (*a*) Discuss briefly the present statistical system in India.

(*b*) Write about National Statistical Commission in India mentioning its *two* important functions.

(*c*) Name *two* Government of India's principal publications each on population and industry. 5½,4,3

7\. (*a*) Obtain the estimated gain in precision due to arbitrary stratification over simple random sampling without replacement.

(*b*) Write short notes on the following :

(*i*) The States' Statistical systems

(*ii*) Economic Census

(*iii*) Objectives of NSSO. 6½,6

8\. (*a*) With two strata, a surveyor would like to have $n_1 = n_2$ for administrative convenience instead of using the values given by Neyman's allocation. If $V(\bar{y}_{st})$ and $V(\bar{y}_{st})_{opt}$ denote the variances of the estimate of population mean under stratified sampling with the condition $n_1 = n_2$ and under Neyman's allocation respectively, then show that the fractional increase in the variance is :

$$\frac{V(\bar{y}_{st}) - V(\bar{y}_{st})_{opt}}{V(\bar{y}_{st})_{opt}} = \left(\frac{r-1}{r+1}\right)^2$$

where $r = n_{1(opt)}/n_{2(opt)}$ and f.p.c. are ignored.

(*b*) Define linear systematic and circular systematic sampling. Prove that systematic sampling is more precise than srswor if the variation within the systematic samples is larger than population variation as a whole. 6½,6

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper	**:**	**6624**	**HC**
Unique Paper Code	:	32351302	
Name of the Paper	:	Group Theory 1	
Name of the Course	:	**B.Sc. (Hons.) Mathematics**	
Semester	:	III	

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.

1. (a) For a fixed point (a, b) in $\mathbb{R}^2$, define $T_{(a,b)} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by $(x, y) \rightarrow (x + a, y + b)$.

Show that $T(\mathbb{R}^2) = \{T_{a,b} \mid a, b \in \mathbb{R}\}$

is a group under function composition. (6)

(b) (i) Find the inverse of $\begin{pmatrix} 2 & 6 \\ 3 & 5 \end{pmatrix}$ in $GL(2, \mathbb{Z}_{11})$. (4)

(ii) Let G be an Abelian group under multiplication with identity e. Show that

$H = \{x^2 | x \in G\}$ is a subgroup of G. (2)

(c) (i) Let G be a group. Show that $Z(G) = \bigcap_{a \in G} C(a)$ where Z(G) is the Center of G and C(a) is the Centralizer of a. (4)

(ii) Let G be the group of nonzero real numbers under multiplication. Show that

$$H = \{x \in G | x = 1 \text{ or } x \text{ is irrational}\}$$

and $K = \{x \in G | x \geq 1\}$ are not subgroups of G. (2)

2. (a) Let G be a group and let $a \in G$. If $|a| = n$, prove that $\langle a \rangle = \{e, a, a^2, ..., a^{n-1}\}$ and $a^i = a^j$ if and only if n divides $i - j$. (6)

(b) Suppose that $|a| = 24$. Find a generator for $\langle a^{21} \rangle \cap \langle a^{10} \rangle$. (6)

(c) If $|a| = n$, show that

$$\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle$$

and that $$|a^k| = \frac{n}{\gcd(n,k)}$$ (6)

3. (a) Define the Alternating Group A_n. Show that it forms a subgroup of the Permutation Group S_n and $|A_n| = \frac{n!}{2}$. (6)

(b) Prove that every group is isomorphic to a group of permutations. (6)

(c) Prove that U(10) is not isomorphic to U(12). (6)

4. (a) State and prove Orbit Stabilizer Theorem. (6½)

(b) (i) Prove that $aH = H$ if and only if $a \in H$. (3)

(ii) Prove that $aH = bH$ or $aH \cap bH = \phi$. (3½)

(c) (i) Prove that order of U(n) is even when $n > 2$. (3)

(ii) Prove that a group of prime order is cyclic. (3½)

5. (a) Let H and K be subgroups of a finite group G and let

$$HK = \{hk | h \in H, k \in K\}$$

and $$KH = \{kh | k \in K, h \in H\}.$$

Prove that HK is a group if and only if HK = KH. (6½)

(b) Let φ be a homomorphism from a group G to a group $\overline{G}$ and let g be an element of G. Prove that

(i) If $\varphi(g) = g'$, then $\varphi^{-1}(g') = \{x \in G | \varphi(x) = g'\} = gKer\varphi$ (4)

(ii) If $|Ker\varphi| = n$, then φ is an n-to-1 mapping from G onto $\varphi(G)$. (2½)

(c) (i) Prove that A_n is normal in S_n. (3½)

(ii) If G is a non-Abelian group of order p^3 (p is prime) and $Z(G) \neq \{e\}$, prove that $|Z(G)| = p$. (3)

6. (a) State and prove The First Isomorphism Theorem. (6½)

(b) Let G be a group and let Z(G) be the center of G. Prove that if $G/Z(G)$ is cyclic, then G is Abelian. (6½)

(c) Let $4Z = (0, \pm 4, \pm 8, \cdots\}$. Find $Z/4Z$. (6½)

This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 6625

Unique Paper Code : 32351303 HC

Name of the Paper : **Multivariate Calculus**

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

Attempt any *five* questions from each Section.

All questions carry equal marks.

Section I

1. Let f be the function defined by :

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x^2 + y^6} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$$

Is f continuous at (0, 0) ? Explain.

P.T.O.

2. Find the equation for each horizontal tangent plane to the surface :

$$z = 5 - x^2 - y^2 + 4y.$$

3. Let f and g be twice differentiable functions of one variable and let $u(x, t) = f(x + ct) + g(x - ct)$ for a constant c. Show that :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

4. Let f have continuous partial derivatives and suppose the maximal directional derivative of f at $P_0(1, 2)$ has magnitude 50 and is attained in the direction from P_0 towards $Q(3, -4)$. Use this information to find $\nabla f(1, 2)$.

5. Find the absolute extrema of $f(x, y) = x^2 + xy + y^2$ on the closed bounded set S where S is the disk $x^2 + y^2 \leq 1$.

6. Find the point on the plane $2x + y + z = 1$ that is nearest to the origin.

Section II

7. Find the area of the region D by setting double integral, where D is bounded by the parabola $y = x^2 - 2$ and the line $y = x$:

8. Write an equivalent integral with the order of integration reversed and then compute the integral :

$$\int_0^4 \int_0^{4-x} xy \, dy dx.$$

9. Calculate the Jacobian of transformation from rectangular to polar coordinates and hence evaluate the integral :

$$\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{1}{\sqrt{9 - x^2 - y^2}} \, dx dy.$$

10. Find the volume V of the solid bounded above by the cylinder $y^2 + z = 4$ and below by $x^2 + 3y^2 = z$.

11. Evaluate the integral below, where D is the region bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$:

$$\iiint_D z \, dx \, dy \, dz.$$

12. Let D be the region in the xy-plane that is bounded by the co-ordinate axes and the line $x + y = 1$. Use the suitable change of variable to compute the integral :

$$\iint_D \left(\frac{x-y}{x+y}\right)^6 dy\,dx.$$

Section III

13. State Green's theorem for simply connected regions. Use Green's theorem to find the work done by the force field $\mathbf{F}(x, y) = (e^x - y^3)\boldsymbol{i} + (\cos y + x^3)\boldsymbol{j}$ along the circle $x^2 + y^2 = 1$ in anticlockwise direction.

14. Give the geometrical interpretation of the surface integral $\iint ds$ over piecewise smooth surface S. Evaluate the surface integral $\iint xz\, ds$ over the surface S which is the part of the plane $x + y + z = 1$ that lies in the first octant.

15. Verify Stokes' theorem for the vector field $\mathbf{F}(x, y, z) = 2z\boldsymbol{i} + 3x\boldsymbol{j} + 5y\boldsymbol{k}$ taking surface σ to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for which $z \geq 0$ with upward orientation and C to be the positively oriented circle $x^2 + y^2 = 4$ that forms the boundary of σ in the xy-plane.

16. State and prove Divergence theorem.

17. Verify that the vector field $\mathbf{F}(x, y) = (e^x \sin y - y)\boldsymbol{i} + (e^x \cos y - x - 2)\boldsymbol{j}$ is conservative using cross partial test. Use a line integral to find the area enclosed by the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

18. Let E be the solid unit cube with opposing corners at the origin and (1, 1, 1) with faces parallel to co-ordinate planes. Let S be the boundary surface of E oriented with the outward pointing normal. If $\mathbf{F}(x, y, z) = 2xy\boldsymbol{i} + 3ye^z\boldsymbol{j} + x \sin z\, \boldsymbol{k}$, find the integral $\iint \mathbf{F}.\mathbf{n}\, ds$ over surface S using divergence theorem.

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6623 **HC**

Unique Paper Code : 32351301

Name of the Paper : Theory of Real Functions

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **three** parts from each question.

1. (a) Use the $\in$-δ definition of the limit to find $\lim_{x\to 2} f(x)$ where $f(x)=\frac{1}{1-x}$. (5)

(b) State and prove Sequential Criterion for Limits. (5)

(c) State Squeeze Theorem. For $n \in N$, $n \geq 3$, derive the inequality, $-x^2 \leq x^n \leq x^2$ for $-1 < x < 1$. Hence prove that $\lim_{x\to 0} x^n = 0$ for $n \geq 3$, assuming that $\lim_{x\to 0} x^2 = 0$. (5)

(d) Let f, g be defined on $A \subseteq R$ to R, and let c be a cluster point of A. Suppose that f is bounded on a neighbourhood of c and that $\lim_{x \to c} g = 0$. Prove that $\lim_{x \to c} fg = 0$. (5)

2. (a) Let $c \in R$ and let f be defined for $x \in (c, \infty)$ and $f(x) \geq 0$ for all $x \in (c, \infty)$. Show that $\lim_{x \to c} f = \infty$ if and only if $\lim_{x \to c} 1/f = 0$. (5)

(b) Prove that

(i) $\lim_{x \to 0} \frac{1}{\sqrt{|x|}} = +\infty, \ x \neq 0$

(ii) $\lim_{x \to 0-} e^{1/x} = 0, \ x \neq 0$. (5)

(c) Let A = R and let f be Dirichlet's function defined by

$$g(x) = \begin{cases} 1, & \text{for } x \text{ rational} \\ -1, & \text{for } x \text{ irrational} \end{cases}$$

Show that f is discontinuous at any point of R. (5)

(d) Let $f : R \to R$ be continuous at c and let $f(c) > 0$. Show that there exists a neighbourhood $V_\delta(c)$ of c such that if $x \in V_\delta(c)$ then $f(x) > 0$. (5)

3. (a) Determine the points of continuity of the function $f(x) = x - [\![x]\!]$, $x \in R$, where $[\![x]\!]$ denotes the greatest integer $n \in \mathbb{Z}$ such that $n \leq x$. (5)

(b) Let $A, B \subseteq R$, let $f: A \to R$ be continuous on A, and let $g : B \to R$ be continuous on B. If $f(A) \subseteq B$, show that the composite function $gof : A \to R$ is continuous on A. (5)

(c) Let f be a continuous real valued function defined on [a, b]. Show that f is a bounded function. (5)

(d) Prove that a polynomial of odd degree has at least one real root. (5)

4. (a) Define uniform continuity of a function on a set $A \subseteq R$. Show that every uniformly continuous function on A is continuous on A. Is the converse true? Justify your answer. (5)

(b) Show that the function $\sqrt{x}$ is uniformly continuous on $[0, \infty)$. (5)

(c) Let I, J be intervals in R, let $g : I \to R$ and $f : J \to R$ be functions such that $f(J) \subseteq I$ and let $c \in J$. If f is differentiate at c and if g is differentiate at f(c), show that the composite function gof is differentiate at c and $(gof)'(c) = g'(f(c)).f'(c)$. (5)

(d) Let $f : R \to R$ be defined by

$$f(x)=\begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x=0 \end{cases}$$

Show that f is differentiable at $x = 0$ and find $f'(0)$. (5)

5. (a) Let f be continuous on [a, b] and differentiate on (a, b). Prove that f is increasing on [a, b] if and only if $f'(x) \geq 0 \ \forall x \in [a, b]$. (5)

(b) State Darboux's Theorem. Suppose that if $f : [0,2] \to R$ is continuous on [0, 2] and differentiate on (0, 2), and that $f(0) = 0$, $f(1) = 1$, $f(2) = 1$.

(i) Show that there exists $c_1 \in (0,1)$ such that $f'(c_1) = 1$

(ii) Show that there exists $c_2 \in (1,2)$ such that $f'(c_2) = 0$

(iii) Show that there exists $c \in (0,2)$ such that $f'(c) = 1/3$. (5)

(c) Find the Taylor series for cos x and indicate why it converges to cos x $\forall x \in R$. (5)

(d) Define a convex function on [a, b]. Check the convexity of the following functions on given intervals :

(i) $f(x) = x - \sin x, \ x \in [0, \pi]$.

(ii) $g(x) = x^3 + 2x, \ x \in [-1, 1]$. (5)